A CAD-Oriented Analytical Model for the Losses of General Asymmetric Coplanar Lines in Hybrid and Monolithic MICs

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***Abstract*—New analytical approximations are derived for the conductor losses of asymmetric coplanar waveguides (ACPW) and coplanar striplines (ACPS) on a finite-thickness dielectric substrate. The expressions hold for lines whose metallizations have thickness much smaller than the slot and strip widths, but suitably larger than the skin penetration depth at the operating frequency. The derivation is based on an extension of the conformal mapping approach formerly proposed by Owyang and Wu [21] for symmetric lines in air. Comparisons with published data from quasistatic or full-wave numerical analyses are presented to validate the expressions derived for both the symmetric and the asymmetric case. The analytical characterization presented in the paper is well suited for inclusion into CAD codes for MMIC design.**

I. Introduction

The MICROSTRIP and the coplanar approach [15], [20] have emerged during the last decade as two alternative approaches to the design of (monolithic) microwave integrated circuits [(M)MICs]. Coplanar (M)MICs have found significant applications, mainly in the area of low-power circuits; moreover, coplanar lines have also been exploited in electro-optical modulators [1]. The interest towards coplanar (M)MICs has fostered the development of CAD-oriented models and, in particular, of analytical approximations for the quasi-TEM parameters of coplanar lines in several configurations (for a review, see [14, Ch. 13] and [10]).

Although the *conductor losses* of coplanar lines have been evaluated numerically through quasistatic [12], [18] of full-wave [17] methods, analytical expressions for the conductor attenuation have been proposed only for symmetric lines. The first analytical model for the conductor losses of symmetric coplanar lines (in air) was published in 1958 by Owyang and Wu [21], well before the pioneering paper by Wen [24] from which the recent history of coplanar lines in MICs originates. In 1979, a different analytical approximation, based on the incremental inductance rule [25], was independently proposed by Gupta, Garg, and Bahl [11], while in 1983 the present author exploited in [7] a trivial extension of Owyang and Wu's formula to dielectric-supported symmetric coplanar lines, also correcting a misprint in the final result of [21], where the conductor attenuation is wrong by a factor 2 (see [21,(21)]). A corrected version of Owyang and Wu's formula also appears in Hoffmann's book in MICs [14]. To the author's knowledge, no extension to asymmetric lines has been presented so far, nor have the available expressions for symmetric lines been validated against the experiment or results from other numerical analysis techniques.

The purpose of the present paper is to develop CAD-oriented, analytical approximations for the conductor attenuation of general, asymmetric coplanar lines. These expressions are derived through an extension of the conformal mapping technique originally applied by Owyang and Wu [21], and hold for lines with metallization thickness much smaller than the strip and slot widths, and in the hypothesis of fully developed skin effect, i.e., when the surface resistance concept applies[1]. The structures considered are shown in Fig. 1(a) (asymmetric coplanar waveguide, ACPW) and in Fig. 1(b) (asymmetric coplanar stripline, ACPS). The ACPW with one lateral ground plane ($ACPW_1$), shown in Fig. 1(c), is a limiting case of both the ACPW and the ACPS. Since the attenuation of dielectric-supported quasi-TEM lines depends on the effective permittivity of the line, the paper also provides a review of this parameter for the structures of interest. Comparisons are carried out on data available from the literature [3], [5], [12], [17], [18] for both the symmetric and the asymmetric case.

The paper is structured as follows. Section II, is devoted to the conformal-mapping analysis of the conductor losses of asymmetric coplanar lines *in vacuo*. Since the analytical treatment involved is rather cumbersome, all results relevant from an applicative standpoint are reported in Section III, where the losses of dielectric-supported lines are discussed. For the asymmetric coplanar waveguide this requires a separate treatment for the effective permittivity of the symmetric line, of the general asymmetric line, and of the line with a single lateral ground plane, while for the asymmetric coplanar stripline a unified treatment can be carried out. Throughout Section III comparisons are presented to validate the relevant expressions.

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[1] A low-frequency limit to the attenuation can be derived, see eg. [5], from the per-unit length dc resistance of the line, which is easily approximated from the geometry. Suitable transition functions can be exploited to blend the low- and high-frequency behavior.

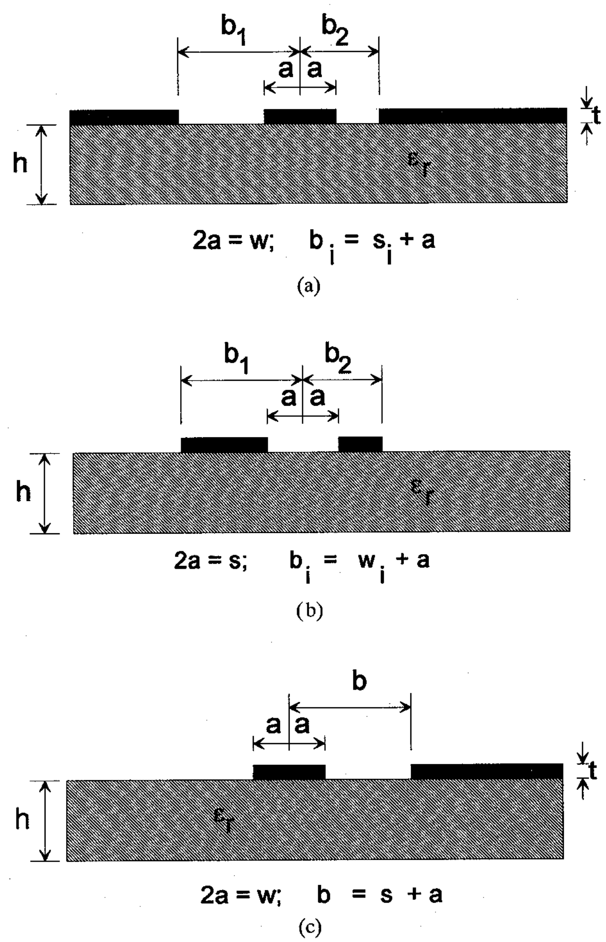

Fig. 1. (a) Asymmetric coplanar waveguide (ACPW): $b_i = s_i + w/2, w = 2a$. (b) Asymmetric coplanar stripline (ACPS): $b_i = w_i + s/2, s = 2a$. (c) Asymmetric coplanar waveguide with one lateral ground plane (ACPW_1): $b = s + w/2, w = 2a$.

II. Evaluating the Losses of Asymmetric Coplanar Lines

In order to evaluate the conductor losses, *in vacuo* coplanar lines will be considered, since the current density distribution, and therefore the power dissipated in the metallization, is not influenced, at least in the quasi-TEM approximation, by the presence of dielectric layers (see eg. [2] and the references therein). From the *in vacuo* parameters (phase velocity $c_0 \approx 3 \times 10^8$ cm/s, characteristic impedance Z_{c0} and conductor attenuation α_{c0}) the corresponding parameters v_f, Z_c, α_c of the dielectric-supported lines can be derived through the line effective permittivity ϵ_{eff} according to the following well-known expressions:

$$v_f = \frac{1}{\sqrt{\epsilon_{\text{eff}}}} c_0 \tag{1}$$

$$Z_c = \frac{1}{\sqrt{\epsilon_{\text{eff}}}} Z_{c0} \tag{2}$$

$$\alpha_c = \alpha_{c0}\sqrt{\epsilon_{\text{eff}}}\,. \tag{3}$$

Moreover, from the effective permittivity the attenuation due to dielectric losses α_d can be expressed according to Welch and Pratt's approach (see e.g. [22]).

The conductor losses are usually evaluated in the skin-effect approximation by means of the *incremental inductance rule* [25]. This technique requires an explicit expression of the line inductance as a function of the line thickness t; asymptotic approximations can be often obtained for $t \to 0$, and the correction with respect to the ideal case $t = 0$ is sometimes expressed via the *equivalent line width* (see eg. [14]). For coplanar lines, a rigorous asymptotic approximation for the thick-line correction is not easily obtained; approximate expressions (see [11] for the symmetric case), though accurate enough when evaluating the line impedance, may fail to provide acceptable values when differentiated with respect to t, as required by the incremental inductance rule. Owyang and Wu's technique [21] circumvents this difficulty by directly evaluating the power dissipated in the line through a conformal mapping approximation of the current density of the finite-thickness structure. In the present paper, this approach is extended to the asymmetric case, although the same result could have been obtained, with a slightly greater analytical effort, through the incremental inductance rule.

The analysis of the asymmetric coplanar waveguide (ACPW, Fig. 1(a)) and the asymmetric coplanar stripline (ACPS, Fig. 1(b)) *in vacuo* will be performed according to the following steps. First, the characteristic parameters of the two structures with zero metallization thickness will be obtained by conformal mapping; then, the conductor attenuation of the ACPS and then of the ACPW will be derived by extending to asymmetric structures the approximate conformal mapping technique described in [21].

A. The Characteristic Parameters of Zero-Thickness Asymmetric Lines in Vacuo

For both the ACPW and the ACPS let us denote the z coordinates of the strip or ground plane edges, from left to right, as z_1, z_2, z_3, z_4. The per-unit-length capacitance of the line can be evaluated by introducing the Schwarz–Christoffel mapping $w(z)$ which transforms the upper part of the z plane (Fig. 2(a)) into the interior of the rectangle in the w plane, as shown in Fig. 2(b). The mapping reads:

$$\begin{aligned} w &= \int_{z_0}^{z} \frac{dw}{dz}\,dz \\ &= A\int_{z_0}^{z} \frac{1}{\sqrt{(z_1 - z)(z_2 - z)(z_3 - z)(z_4 - z)}}\,dz \end{aligned} \tag{4}$$

where A is an (arbitrary) scale factor. The length of the sides of the rectangle in the w plane are:

$$\begin{aligned} |w(z_3) - w(z_4)| &= |w(z_2) - w(z_1)| \\ &= \left|\int_{z_3}^{z_4} \frac{dw}{dz}\,dz\right| \\ &= \frac{2}{\sqrt{(z_4 - z_2)(z_3 - z_1)}}\,K(k') \end{aligned} \tag{5}$$

$$\begin{aligned} |w(z_2) - w(z_3)| &= |w(z_4) - w(z_1)| \\ &= \left|\int_{z_2}^{z_3} \frac{dw}{dz}\,dz\right| \end{aligned}$$

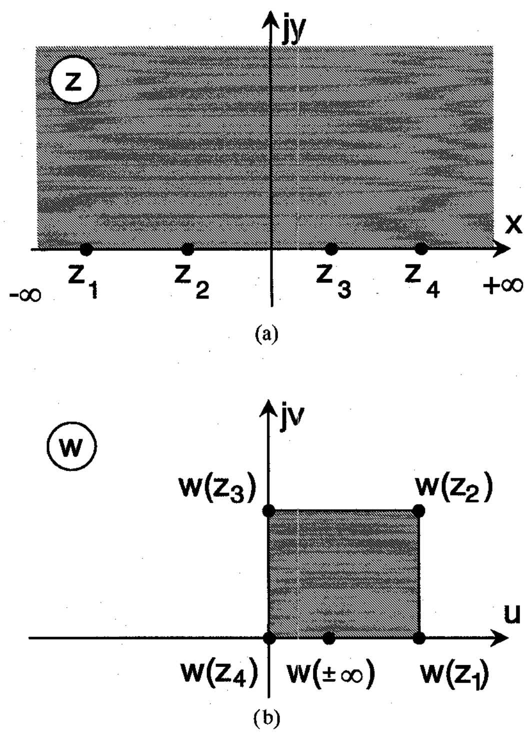

Fig. 2. Conformal mapping for zero-thickness ACPS. (a) Original z-plane. (b) Transformed w-plane.

$$= \frac{2}{\sqrt{(z_4 - z_2)(z_3 - z_1)}} K(k) \tag{6}$$

where K is the complete elliptic integral of the first kind, and the arguments k and k' read

$$k = \sqrt{\frac{(z_3 - z_2)(z_4 - z_1)}{(z_4 - z_2)(z_3 - z_1)}} \tag{7}$$

$$k' = \sqrt{1 - k^2} = \sqrt{\frac{(z_4 - z_3)(z_2 - z_1)}{(z_4 - z_2)(z_3 - z_1)}}. \tag{8}$$

Therefore the per-unit-length *in vacuo* capacitances of the two asymmetric lines are

$$C_0^{\text{ACPW}} = 2\epsilon_0 \frac{|w(z_2) - w(z_3)|}{|w(z_2) - w(z_1)|} = 2\epsilon_0 \frac{K(k)}{K(k')} \tag{9}$$

$$C_0^{\text{ACPS}} = 2\epsilon_0 \frac{|w(z_3) - w(z_4)|}{|w(z_2) - w(z_3)|} = 2\epsilon_0 \frac{K(k')}{K(k)} \tag{10}$$

where ϵ_0 is the *in vacuo* permittivity. Taking into account that the per-unit-length inductance of the line is $L = 1/(c_0^2 C_0)$, where C_0 is the *in vacuo* per-unit-length capacitance, the characteristic impedance *in vacuo* $Z_{c0} = \sqrt{L/C_0}$ is, for the two structures

$$Z_{c0}^{\text{ACPW}} = 60\pi \frac{K(k')}{K(k)} \tag{11}$$

$$Z_{c0}^{\text{ACPS}} = 60\pi \frac{K(k)}{K(k')}. \tag{12}$$

B. The Conductor Losses of Finite-Thickness Lines in Vacuo

We shall first consider in detail in Section II-B-1 the case of the ACPS; the treatment of the ACPW, which is fully analogous, will be summarized in Section II-B-2. If the metallization thickness is suitably larger than the skin penetration depth, the attenuation due to conductor losses can be given the well-known expression [4]

$$\alpha_{c0} = \frac{R_s}{2 Z_{c0} I^2} \oint_\gamma |J|^2 \, dl \tag{13}$$

where R_s is the surface resistance, I the total rms current carried by the line, J, the longitudinal current density on the line, γ the conductor periphery. For the sake of brevity, the line integral in (13) will be referred to as the *loss factor.*

Before evaluating the loss factor for the ACPS and ACPW, the current density must be normalized. In lines with thin metallizations, this can be (approximately) done on the zero thickness structure, for which

$$J(z) = \pm \tilde{I} \left| \frac{1}{\sqrt{(z_1 - z)(z_2 - z)(z_3 - z)(z_4 - z)}} \right| \tag{14}$$

for both the ACPS and the ACPW; $\tilde{I}$ is a normalization constant, taken as positive. For the ACPW, the current density is zero on the intervals $[z_1, z_2]$ and $[z_3, z_4]$ and takes opposite signs on the central strip and lateral ground planes, while for the complementary ACPS the current density is zero on the intervals $[-\infty, z_1], [z_2, z_3], [z_4, \infty]$ and takes opposite signs on the two strips. Integration of J on the central strip (for the ACPW) or on either strips (for the ACPS) yields

$$I^{\text{ACPW}} = 2 \int_{z_2}^{z_3} |J||dz| = \frac{4 \tilde{I}^{\text{ACPW}}}{\sqrt{(z_4 - z_2)(z_3 - z_1)}} K(k') \tag{15}$$

$$I^{\text{ACPS}} = 2 \int_{z_1}^{z_2} |J||dz| = \frac{4 \tilde{I}^{\text{ACPS}}}{\sqrt{(z_4 - z_2)(z_3 - z_1)}} K(k). \tag{16}$$

The factor 2 accounts for the current flowing on the upper and lower sides of metallizations. Thus, for both structures

$$Z_{c0} I^2 = \frac{960 \pi \tilde{I}^2}{(z_4 - z_2)(z_3 - z_1)} K(k) K(k'). \tag{17}$$

This result will be used in the following analysis.

1) The conductor losses of the ACPS: To estimate the current density on the finite-thickness structure a two-step conformal mapping can be exploited, as done in [21] for a symmetric CPS. The mapping, shown in Fig. 3, transforms the upper half of the original structure (ζ plane) into the upper half of a zero-thickness structure (z plane) and finally into the interior of a rectangle (w plane).

The mapping $w(z)$ is the same used for the zero-thickness structure, while the mapping $z(\zeta)$ can be obtained through the Schwarz–Christoffel technique. Let us define

$$\zeta_i' = \zeta_i + j\tau, \qquad i = 1, \cdots, 4 \tag{18}$$

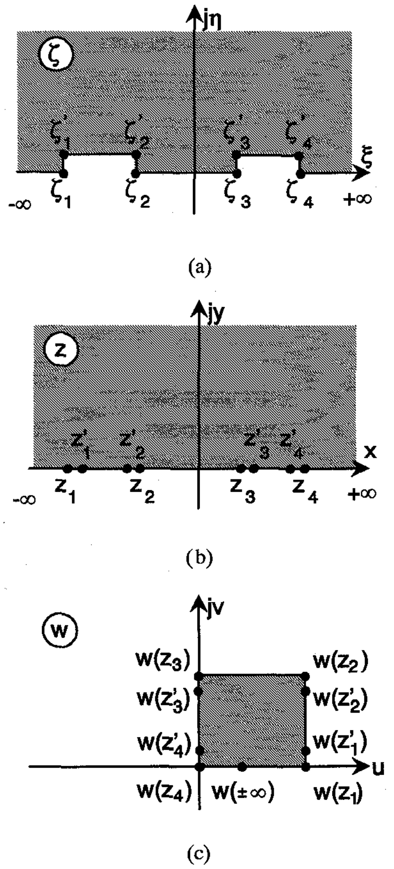

Fig. 3. Conformal mapping for finite metallization thickness ACPS. (a) Original ζ-plane. (b) Intermediate z-plane (zero-thickness ACPS). (c) Transformed w-plane.

where $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ are the ζ coordinates of the strip edges (see Fig. 3) and, for expediency, the strip half-thickness $\tau = t/2$ is used instead of t. The images in the z plane will be $z_i = z(\zeta_i), z_i' = z(\zeta_i')$, where the mapping $\zeta(z)$ is expressed as

$$\zeta = \int_{\zeta_0}^{\zeta} d\zeta = \int_{z_0}^{z} \frac{d\zeta}{dz}\, dz$$
$$= B \int_{z_0}^{z} \sqrt{\frac{(z_1' - z)(z_2' - z)(z_3' - z)(z_4' - z)}{(z_1 - z)(z_2 - z)(z_3 - z)(z_4 - z)}}\, dz \quad (19)$$

and B is a scale factor.

The explicit determination of the mapping (19) is not needed for further analysis according to Owyang and Wu's technique, at least in the thin strip approximation. If fact, for small τ (i.e., if $\tau \ll |\zeta_2 - \zeta_1|, \tau \ll |\zeta_3 - \zeta_2|, \tau \ll |\zeta_4 - \zeta_3|$), one has $\zeta_1 \approx \zeta_1', \zeta_2 \approx \zeta_2', \zeta_3 \approx \zeta_3', \zeta_4 \approx \zeta_4'$ and thus $z_1 \approx z_1', z_2 \approx z_2', z_3 \approx z_3', z_4 \approx z_4'$. Thus, with a proper choice of the scale factor B, $d\zeta/dz \approx 1$ and therefore $\zeta \approx z$ apart from a small neighborhood of the strip edges. Therefore, if ζ is not close to the jth edge, one can set $\zeta - \zeta_j \approx z - z_j \approx z - z_j'$. To proceed with the analysis, we only need to relate the strip thickness in the original ζ plane to the differences $z_i' - z_i, i = 1, \cdots, 4$, in the transformed z plane. To this aim we notice that, for $z_1 < z < z_1'$ and within the approximations outlined above, one has

$$\frac{d\zeta}{dz} = \sqrt{\frac{z_1' - z}{z - z_1}}. \quad (20)$$

Similar expressions hold for the other strip sides. Therefore, by integrating (20) along the (half) strip side, one has

$$j\tau = \int_{\zeta_1}^{\zeta_1 + j\tau} d\zeta = \int_{z_1}^{z_1'} \frac{d\zeta}{dx}\, dz$$
$$\approx \int_{z_1}^{z_1'} \sqrt{\frac{z_1' - z}{z - z_1}}\, dz = j\frac{\pi}{2}(z_1' - z_1) \quad (21)$$

and similarly for the other strip sides, thereby leading to the result

$$z_1' - z_1 = 2\tau/\pi, \quad z_2 - z_2' = 2\tau/\pi, \quad z_3' - z_3 = 2\tau/\pi, \quad z_4 - z_4' = 2\tau/\pi \quad (22)$$

which will be exploited later on.

Evaluation of the loss factor: For the ACPS the loss factor in (13) can be expressed as

$$\oint_\gamma |J|^2\, dl = 2\left(\int_{\zeta_1}^{\zeta_2} J^2(\zeta)|d\zeta| + \int_{\zeta_3}^{\zeta_4} J^2(\zeta)|d\zeta|\right). \quad (23)$$

The two integrals can be evaluated in the z plane as follows. Let us express the current density through the uniform density in the w plane $\tilde{I}^{\rm ACPS}$ and the scale factors $|dw/dz|$ and $|dz/d\zeta|$ as [21]

$$J(\zeta) = \tilde{I}^{\rm ACPS}\left|\frac{dw}{dz}\right|\left|\frac{dz}{d\zeta}\right| \quad (24)$$

where $\tilde{I}^{\rm ACPS}$ is related through (16) to the total strip current, and $|dw/dz|$, $|d\zeta/dz|$ are defined in (4) and (19), respectively. Therefore one has

$$\oint_\gamma |J|^2\, dl = 2\left(\tilde{I}^{\rm ACPS}\right)^2\left(\int_{z_1}^{z_2} P(z)|dz| + \int_{z_3}^{z_4} P(z)|dz|\right) \quad (25)$$

where $P(z)$ is defined in (26) (see bottom of page). The two integrals in the right-hand side of (25) can be interpreted as the *normalized loss factors* for the left- and right-hand strip, respectively. As shown in Appendix A, the normalized loss factors can be approximated, for lines whose

$$P(z) = \left|\frac{1}{\sqrt{(z - z_1)(z - z_1')(z - z_2)(z - z_2')(z - z_3)(z - z_3')(z - z_4)(z - z_4')}}\right|. \quad (26)$$

metallizations have thickness much smaller than the slot and strip widths, as

$$\begin{aligned}\int_{z_1}^{z_2} P(z)|dz| \approx\ & \frac{1}{(\zeta_2-\zeta_1)(\zeta_3-\zeta_1)(\zeta_4-\zeta_1)} \cdot \left\{\log\left[\frac{2\pi}{\tau}(\zeta_2-\zeta_1)\right]+\pi\right\} \\ & + \frac{1}{(\zeta_2-\zeta_1)(\zeta_3-\zeta_2)(\zeta_4-\zeta_2)} \cdot \left\{\log\left[\frac{2\pi}{\tau}(\zeta_2-\zeta_1)\right]+\pi\right\} \\ & + \frac{1}{(\zeta_3-\zeta_1)(\zeta_3-\zeta_2)(\zeta_4-\zeta_3)} \cdot \log\left(\frac{\zeta_3-\zeta_2}{\zeta_3-\zeta_1}\right) \\ & + \frac{1}{(\zeta_4-\zeta_1)(\zeta_4-\zeta_2)(\zeta_4-\zeta_3)} \cdot \log\left(\frac{\zeta_4-\zeta_1}{\zeta_4-\zeta_2}\right), \end{aligned} \tag{27}$$

$$\begin{aligned}\int_{z_3}^{z_4} P(z)|dz| \approx\ & \frac{1}{(\zeta_4-\zeta_3)(\zeta_4-\zeta_2)(\zeta_4-\zeta_1)} \cdot \left\{\log\left[\frac{2\pi}{\tau}(\zeta_4-\zeta_3)\right]+\pi\right\} \\ & + \frac{1}{(\zeta_4-\zeta_3)(\zeta_3-\zeta_2)(\zeta_3-\zeta_1)} \cdot \left\{\log\left[\frac{2\pi}{\tau}(\zeta_4-\zeta_3)\right]+\pi\right\} \\ & + \frac{1}{(\zeta_4-\zeta_2)(\zeta_3-\zeta_2)(\zeta_2-\zeta_1)} \cdot \log\left(\frac{\zeta_3-\zeta_2}{\zeta_4-\zeta_2}\right) \\ & + \frac{1}{(\zeta_4-\zeta_1)(\zeta_3-\zeta_1)(\zeta_2-\zeta_1)} \cdot \log\left(\frac{\zeta_4-\zeta_1}{\zeta_3-\zeta_1}\right), \end{aligned} \tag{28}$$

respectively.

Attenuation: By substituting (27) and (28) into (25) the overall loss factor is obtained. The conductor loss attenuation of the ACPS can be derived from (13) by expressing $Z_{c0}I^2$ according to (17), with the approximations $z_i - z_j \approx \zeta_i - \zeta_j, i, j = 1, \cdots, 4$. After elementary algebraic manipulations, the ACPS attenuation can be put in the following convenient form:

$$\begin{aligned}\alpha_{c0}^{\mathrm{ACPS}} = & \frac{R_s}{480\pi K(k)K(k')} \cdot \left[\Phi(\zeta_2-\zeta_1,k) - \Phi(\zeta_4-\zeta_1,k') \right. \\ & \left. + \Phi(\zeta_3-\zeta_2,k') + \Phi(\zeta_4-\zeta_3,k)\right] \end{aligned} \tag{29}$$

where

$$\begin{aligned}\Phi(\zeta_i-\zeta_j,\kappa) &= \frac{1}{\zeta_i-\zeta_j} \cdot \left\{\log\left[\frac{2\pi(\zeta_i-\zeta_j)\kappa}{\tau}\right]+\pi\right\} \\ &= \frac{1}{\zeta_i-\zeta_j}\left\{\log\left[\frac{4\pi(\zeta_i-\zeta_j)\kappa}{t}\right]+\pi\right\}\end{aligned} \tag{30}$$

in which the strip thickness t has been again introduced.

2) The Conductor Losses of the ACPW: The treatment of the ACPW is similar to the one of the ACPS. By use of the same two-step conformal mapping method as for the ACPS, the loss factor in (13) can now be expressed as

$$\oint_\gamma J^2\, dl = 2\left(\int_{-\infty}^{\zeta_1} J^2(\zeta)|d\zeta| + \int_{\zeta_2}^{\zeta_3} J^2(\zeta)|d\zeta| + \int_{\zeta_4}^{\infty} J^2(\zeta)|d\zeta|\right). \tag{31}$$

On reducing the integration to the z plane, the following integrals have to be evaluated:

$$\int_{-\infty}^{z_1} P(z)|dz|, \qquad \int_{z_2}^{z_3} P(z)|dz|, \qquad \int_{z_4}^{\infty} P(z)|dz| \tag{32}$$

where $P(z)$ is again given by (26). Finally, one obtains for the conductor loss attenuation of the ACPW the following expression, which holds if the strip thickness is much smaller than the slot widths $|\zeta_2 - \zeta_1|$ and $|\zeta_4 - \zeta_3|$ and the strip width $|\zeta_3 - \zeta_2|$

$$\begin{aligned}\alpha_{c0}^{\mathrm{ACPW}} = & \frac{R_s}{480\pi K(k)K(k')} \cdot \left[\Phi(\zeta_2-\zeta_1,k) - \Phi(\zeta_4-\zeta_1,k') \right. \\ & \left. + \Phi(\zeta_3-\zeta_2,k') + \Phi(\zeta_4-\zeta_3,k)\right] \end{aligned} \tag{33}$$

where Φ is given again by (30). Comparison of (29) and (33) reveals that, at least within the approximations exploited in deriving such expressions, *in vacuo* complementary coplanar lines, i.e., an ACPW and an ACPS having the same geometry, have the same attenuation. This result is an obvious extension of Owyang and Wu's theory [21], in which the same property is shown in hold for symmetric lines.

III. The Conductor Losses of Dielectric-Supported Coplanar Lines

The conductor losses of dielectric-supported coplanar lines also depend, according to (3), on the effective permittivity of the line. For this parameter ax exact quasi-TEM expression exists only for lines supported by semi-infinite dielectric layers of relative permittivity ϵ_r, for which $\epsilon_{\mathrm{eff}} = (1+\epsilon_r)/2$.

Approximations to the ϵ_{eff} of symmetric coplanar lines supported by finite-extent dielectric substrates have been derived [8], [14], [23] through the so-called *method of superposition of partial capacitances* (see e.g. [14]). As a rule of thumb, this approximation is satisfactory for symmetric lines when the substrate thickness is larger than half of the overall lateral extension of the line $b_1 + b_2$ (see Fig. 1)[2]. For moderately asymmetric lines, one may postulate a similar empirical rule, thus requiring $h > b_{\max}$ where $b_{\max} = \max(b_1, b_2)$. For

[2] This rather conservative estimate is based on the discussion presented in [8].

strongly asymmetric lines, e.g. for the ACPW_1, it may be surmised that the field lines crossing the narrower slot dominate the effective permittivity, thereby leading to the requirement $h > b_{\min}$ where $b_{\min} = \min(b_1, b_2)$, which becomes $h > b$ for the ACPW_1.

Concerning the asymmetric structures considered in the paper, the partial capacitance approach can be implemented exactly in terms of elliptic integrals for the symmetric coplanar waveguide (CPW) [8], [14], [23], the ACPW with one lateral ground plane (ACPW_1) [14] and the general (symmetric and asymmetric) ACPS [14]. For the general ACPW the exact implementation leads to expressions in terms of hyperelliptic integrals, to be evaluated numerically [9], [19]. An approximate expression for the effective permittivity of the ACPW in terms of elliptic integrals was proposed in [6]. In Section III–A, the expression in [6] is compared to the exact, numerical implementation of the partial capacitance approach, so as to identify, at least for GaAs substrates, the limits of its applicability. Further comparisons with data derived from numerical approaches suggest that, for the ACPW on a finite-thickness dielectric substrate, the accuracy achieved by the present analytical model, coupled to the expressions of the effective permittivity in [6], is likely to be adequate in estimating the line losses and the characteristic impedance, while for the effective permittivity the partial capacitance approximation may be inadequate for the design of frequency-selective components (eg. filtering sections, electro-optical modulators).

A. General ACPW

From (33) and (3) the conductor loss attenuation for a general dielectric-supported ACPW reads, in natural units (Np/m):

$$\alpha_c^{\text{ACPW}} = \frac{R_s\sqrt{\epsilon_{\text{eff}}^{\text{ACPW}}}}{480\pi K(k)K(k')} \cdot \left[\Phi(b_1 - a, k) + \Phi(b_2 - a, k) + \Phi(2a, k') - \Phi(b_1 + b_2, k')\right] \tag{34}$$

where $\epsilon_{\text{eff}}^{\text{ACPW}}$ is the effective permittivity of the line, K is the complete elliptic integral of the first kind, R_s is the surface resistance, Φ is defined in (30), and the parameters k and k' can be suitable expressed as a function of the geometrical parameters of the line (see Fig. 1(a)) as

$$k = \sqrt{\frac{2a(b_1 + b_2)}{(b_1 + a)(b_2 + a)}} \tag{35}$$

$$k' = \sqrt{1 - k^2} = \sqrt{\frac{(b_1 - a)(b_2 - a)}{(b_1 + a)(b_2 + a)}}\,. \tag{36}$$

Equation (34) holds for lines in which the metallization is thin, i.e., t is much smaller than the strip width w and the slot widths s_1 and s_2 but suitably larger than the skin penetration depth. This condition is verified in most low-loss lines for (M)MICs, while MMIC interdigitated structures often have strip aspect ratios of the order of the unity. Similarly, also in coplanar lines for electro-optical modulators [1] the slot width is often limited to a few μm in order to achieve satisfactory coupling to the optical waveguide, and therefore metallizations cannot be considered as thin.

The evaluation of the effective permittivity of asymmetric coplanar waveguides on a finite-thickness dielectric substrate was addressed in [6] through the partial capacitance approach. However, the conformal mapping exploited therein to compute the layer capacitances is approximate, and only yields the exact result for symmetric lines, and in the limit

$$\Delta s \to 1, \qquad h/(b_1 + b_2) > 0 \tag{37}$$

where the offset parameter Δs is defined as

$$\Delta s = \frac{|s_1 - s_2|}{s_1 + s_2} = \frac{|b_1 - b_2|}{b_1 + b_2 - 2a}\,. \tag{38}$$

In this case, in fact, the width of either slot vanishes and correspondingly the effective permittivity of the lower half-space is dominated by the layer permittivity ϵ_r, thereby making $\epsilon_{\text{eff}}^{\text{ACPW}} \to (\epsilon_r + 1)/2$. The approximation implicit in the conformal mapping technique of [6] can be clearly seen in [6, Fig. 2(a) and (b)], where a magnetic wall is introduced along the negative imaginary axis on the interval $[-j, -j\infty]$ by transforming the bottom of the dielectric layer. The magnetic wall can be removed without influencing the field lines, as done in Fig. 2(c), only when its presence is already implicit in the structure for symmetry, as in the symmetric CPW.

The effective permittivity which derives from the approximate conformal mapping of [6] can be expressed in a more compact way than it was done in the original reference as

$$\epsilon_{\text{eff}}^{\text{ACPW}} \approx 1 + \frac{\epsilon_r - 1}{2}\frac{K(k_2')K(k)}{K(k_2)K(k')} \tag{39}$$

where k_2 is defined in (40), (see bottom of page), $k_2' = \sqrt{1 - k_2^2}$ and k is defined in (35).

To estimate the accuracy of (39), the partial capacitance technique has been implemented exactly through the partly numerical conformal mapping method described in [9], [19] and the effective permittivity of lines on GaAs substrates has been computed as a function of the offset index Δs for several values of the substrate thickness and central strip width. An example of the behavior of the effective permittivity as a function of the offset parameter for the two models is given in Fig. 4. For GaAs substrates, the relative error of (39), which increases with decreasing strip width, can be estimated to be

$$k_2 = \sqrt{\frac{2\sinh(\pi a/2h)[\sinh(\pi b_2/2h) + \sinh(\pi b_1/2h)]}{[\sinh(\pi a/2h) + \sinh(\pi b_1/2h)][\sinh(\pi a/2h) + \sinh(\pi b_2/2h)]}} \tag{40}$$

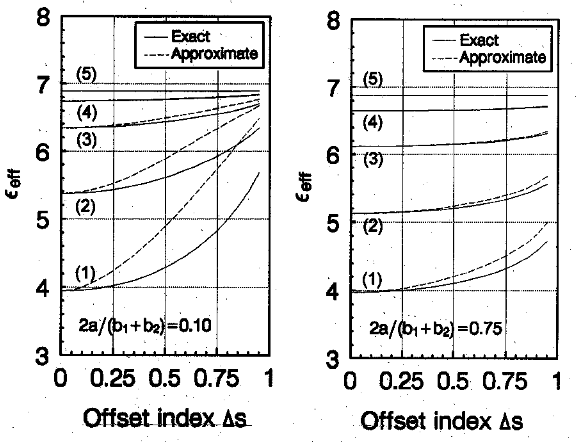

Fig. 4. Example of behavior of the effective permittivity of ACPW of finite-extent GaAs substrate ($\epsilon_r = 12.8$) as a function of Δs, for narrow ($2a/(b_1 + b_2) = 0.1$) and wide ($2a/(b_1 + b_2) = 0.75$) lines. The ratio $\chi = h/(b_1 + b_2)$ is: (1)—$\chi = 1/8$; (2)—$\chi = 1/4$; (3)—$\chi = 1/2$; (4)—$\chi = 1$; (5)—$\chi = 2$. The continuous line is the exact implementation of the partial capacitance approach (see text), the dotted line (39) [6].

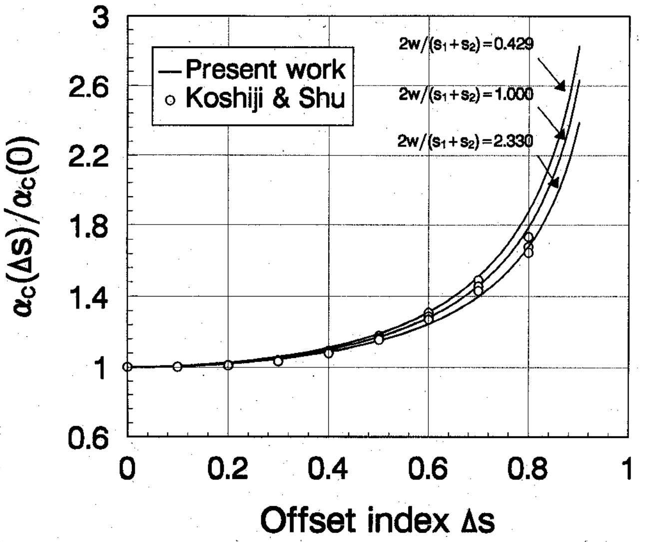

Fig. 5. Normalized conductor loss $\alpha_c(\Delta s)/\alpha_c(0)$ of ACPW on finite-extent substrate ($\epsilon_r = 2.7$) as a function of the offset index Δs, for several values of the ratio $w/(s_1 + s_2)$. The ratio between the conductor thickness and the ground-to-ground spacing is $t/(w + s_1 + s_2) = 0.0083$; the normalized substrate thickness is $h/(w+s_1+s_2) = 0.5$. Circles are from the quasi-static numerical approach of [18] (see [18, Fig. 6(a), (b), (c)]); the continuous line is from the present approach.

lower than 5% for any Δs on the useful impedance range (eg. for $2a/(b_1 + b_2) > 0.1$) when the ratio between the substrate thickness h and the total slot width $b_1 + b_2$ is greater than $1/2$.

Comparisons have been carried out with data on lossy ACPWs on finite-thickness substrates derived in [18] through a quasi-static approach and in [17] through a full-wave method; in both analyses the strip thickness is accounted for when evaluating the line impedance and effective permittivity. In Fig. 5 the behavior of the normalized conductor attenuation from (34) as a function of the offset index is compared with the results in [18] (34); the agreement found is good.

In Figs. 6, 7, and 8 the quasi-static characterization of the asymmetric coplanar waveguide on a finite-extent GaAs substrate is compared with the full-wave results of [17]. In

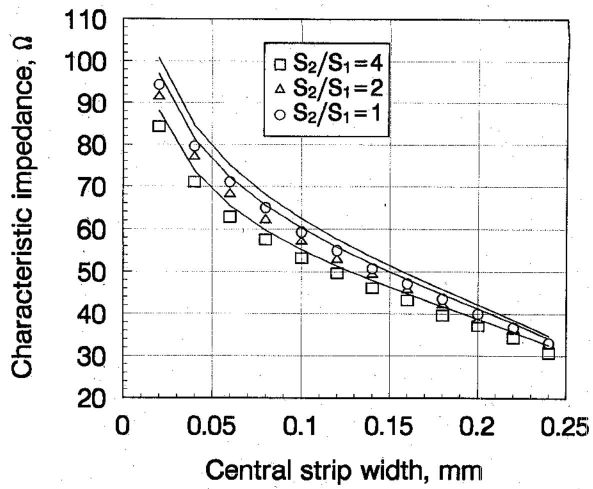

Fig. 6. Characteristic impedance of ACPW as a function of the central strip width, for several values of the asymmetry parameter s_1/s_2. The symbols are data from [17], accounting for finite metallization thickness; the continuous line is from the present approach (zero-thickness line). The substrate thickness is $h = 0.1$ mm, the permittivity $\epsilon_r = 12.8$; the spacing between the lateral ground plane is $b_1 + b_2 = 0.3$ mm; the metallization thickness is $t = 3\mu$m; the frequency $f = 60$ GHz.

order to correctly appreciate this comparison, it should be considered that in evaluating the line parameters a zero-thickness, quasi-static approximation has been used, while the full-wave analysis is frequency dependent, and takes into account the finite conductor thickness. Moreover, since $h/(b_1 + b_2) = 1/3$, the partial capacitance approach and the approximation of (39) are not expected to be overly accurate. Indeed, although the qualitative trend of ϵ_{eff} as a function of the line asymmetry is preserve, see Fig. 7, the quasi-static approximation on the infinitely thin line fails to yield the fairly complex functional dependence displayed by the results from [17].[3] Nevertheless, the effect of such discrepancies on the line impedance is less significant, and a better agreement can be seen in Fig. 6, although the results from [17] show a lower value, presumably because of the effect of the finite-thickness metallization. In order to separate the effect of the slightly different quasi-static parameters from the effect of the line loss per se, the total line attenuation has been plotted in Fig. 8 as a function of the line impedance. The agreement is fairly good also for large asymmetries.

In conclusion, the expressions presented for the conductor losses of the ACPW on finite-thickness substrates yield results close to the ones derived from more accurate numerical approaches. On the other hand, the use of (39) for the effective permittivity should be restricted to thick substrates, and is not expected to be accurate enough for the design of frequency-selective components. Finally, since Z_c depends on ϵ_{eff} through a square root, its sensitivity the exact value of ϵ_{eff} is somewhat lower. The satisfactory comparison presented in [6] between the analytical approximation and experimental data concerning lines on allumina substrates could suggest that, since the approximate expression slightly overestimates the

[3]The exact implementation of the partial capacitance technique yields slightly lower values of ϵ_{eff} but does not lead to a substantially different behavior.

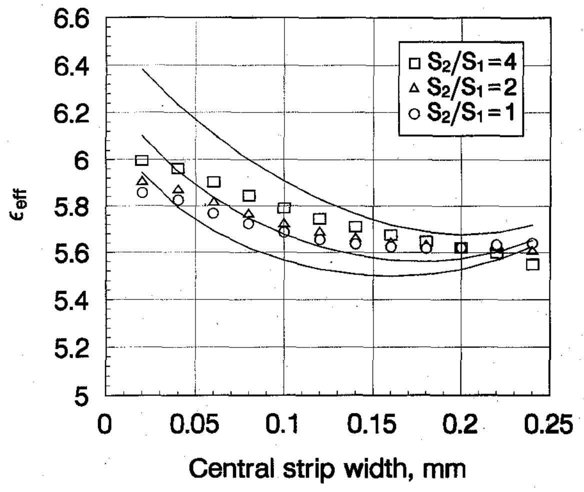

Fig. 7. Effective permittivity of ACPW as a function of the central strip width, for several values of the asymmetry parameter s_1/s_2. The symbols are data from [17], accounting for finite metallization thickness; the continuous line is from the present approach (zero-thickness line). The substrate thickness is $h = 0.1$ mm, the permittivity $\epsilon_r = 12.8$; the spacing between the lateral ground planes is $b_1 + b_2 = 0.3$ mm; the metallization thickness is $t = 3\mu$m; the frequency $f = 60$ GHz.

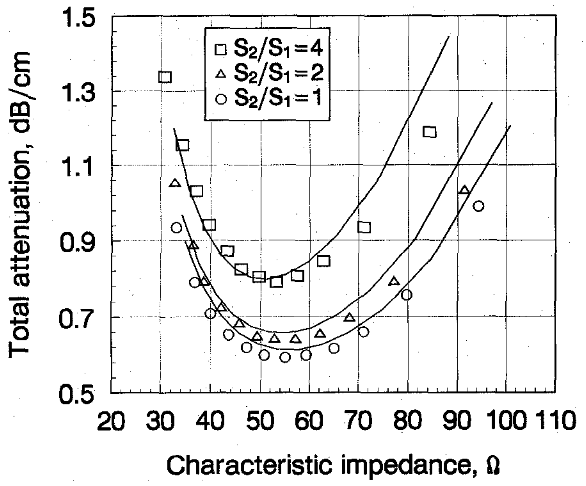

Fig. 8. Total attenuation of ACPW as function of the line impedance, for several values of the asymmetry parameter s_1/s_2. The symbols are data from [17]; the continuous line is from the present approach. The substrate thickness is $h = 0.1$ mm, the permittivity $\epsilon_r = 12.8$; the spacing between the lateral ground planes is $b_1 + b_2 = 0.3$ mm; the frequency is $f = 60$ GHz, the metallization thickness is $t = 3\mu$m, the loss tangent $\tan\delta = 0.0006$. A copper metallization was assumed.

effective permittivity, it partly compensates for the effect of both the dispersion and the finite metallization thickness.

B. Symmetric CPW

For a symmetric coplanar waveguide supported by a dielectric layer of thickness h one has $b_1 = b_2 = b$. Taking into account the following transformation rules for elliptic integrals:

$$K(k) = (1 + k_s)K(k_s) \tag{41}$$

$$K(k') = \frac{1 + k_s}{2} K(k_s') \tag{42}$$

where $k_s' = \sqrt{1 - k_s^2}$ and

$$k = \frac{2\sqrt{k_s}}{1 + k_s} \tag{43}$$

$$k' = \frac{1 - k_s}{1 + k_s} \tag{44}$$

for $b_1 = b_2 = b, k_s = a/b$ the general expression for the attenuation given in (34) can be shown to reduce to Owyang and Wu's formula [7], [14], [21], which is reported here for completeness

$$\alpha_c^{\text{CPW}} = \frac{R_s\sqrt{\epsilon_{\text{eff}}^{\text{CPW}}}}{480\pi K(k_s)K(k_s')(1 - k_s^2)} \times \left\{ \frac{1}{a}\left[\pi + \log\left(\frac{8\pi a(1 - k_s)}{t(1 + k_s)}\right)\right] + \frac{1}{b}\left[\pi + \log\left(\frac{8\pi b(1 - k_s)}{t(1 + k_s)}\right)\right]\right\}. \tag{45}$$

The effective permittivity can be approximated through an exact application of the method of partial capacitances as [8], [14], [23]

$$\epsilon_{\text{eff}}^{\text{CPW}} = 1 + \frac{\epsilon_r - 1}{2}\frac{K(k_s')}{K(k_s)}\frac{K(k_1)}{K(k_1')} \tag{46}$$

where

$$k = \frac{\sinh(\pi a/2h)}{\sinh(\pi b/2h)} \tag{47}$$

and $k_1' = \sqrt{1 - k_1^2}$.

In Fig. 9 the normalized attenuation derived from Owyang and Wu's formula (45) is compared to the results obtained by Gopinath [12] through a quasi-static numerical approach, and to the analytical approximation proposed by Gupta, and Bahl [11]. The agreement between Owyang and Wu's formula and Gopinath's results is good, while Gupta's expression leads to a fairly higher attenuation for $a/b > 0.4$, though presenting the same qualitative behavior as a function of the line parameters. Although both Owyang and Wu's approach and Gupta, Garg, and Bahl's are consistent with the incremental inductance rule, the behavior of the line impedance as a function of the line thickness as postulated in [11] is based on an approximation which may be not accurate enough when differentiated with respect to the line thickness.

More recently, the evaluation of the attenuation of CPWs has been addressed by Jackson [16] and by Kitazawa and Itoh [17] through a full-wave technique; both approaches are based on the surface resistance approximation. Fig. 10 shows the total attenuation $\alpha_c + \alpha_d$ of lines on a GaAs substrate as a function of the line impedance, for several values of the total slot width $2b$. The agreement between the analytical approach and the numerical results is particularly good for the data from [17]. For the parameters in Fig. 10 the dielectric attenuation is much lower than the conductor loss, though not completely negligible.

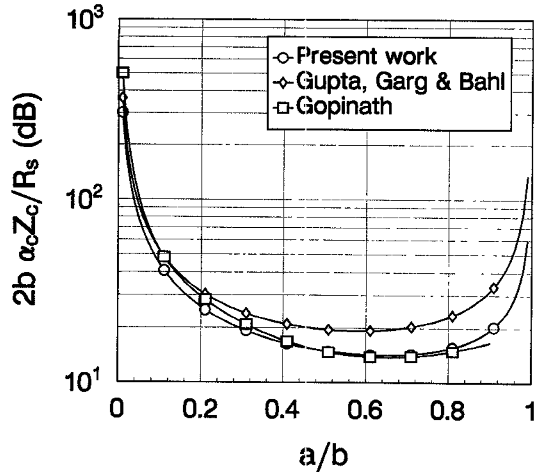

Fig. 9. Normalized attenuation for symmetric CPW as a function of a/b: comparison between (45) [21], Gopinath's quasi-static numerical approach [12] and the analytical approximation of Gupta, Garg, and Bahl [11]. The metallization thickness is $t = 5\mu m$ [13] and $2b = 1.2$ mm.

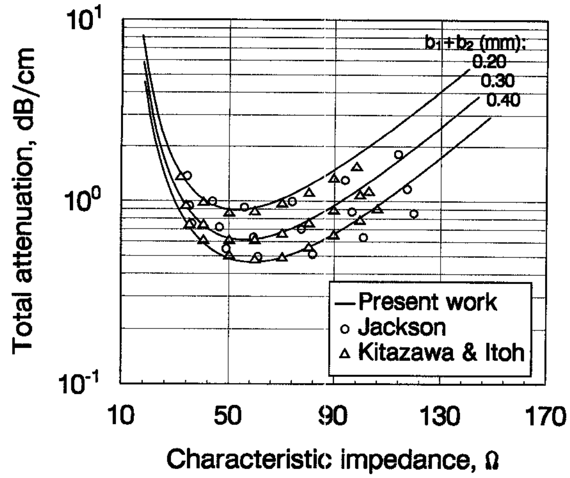

Fig. 10. Total loss of symmetric coplanar waveguide on finite-extend GaAs substrate ($\epsilon_r = 12.8, \tan\delta = 0.0006$) as a function of Z_c for several values of $2b$, according to Jackson [16], Kitazawa and Itoh [17] and to (45) [21]. The substrate thickness is $h = 100\mu m$, the metallization thickness $t = 3\mu m$; the operating frequency is 60 GHz. For the surface resistivity the standard value for copper has been assumed ($R_s = 8.24 \times 10^{-3}\sqrt{f_{\mathrm{GHz}}}\,\Omega$, [14]).

C. ACPW with One Lateral Ground Plane

The conductor loss of the ACPW with one lateral ground plane (ACPW$_1$) can be derived from (34) by taking the limit $b_1 \to \infty$ and letting $b_2 = b$ as

$$\alpha_c^{\mathrm{ACPW_1}} = \frac{R_s\sqrt{\epsilon_{\mathrm{eff}}^{\mathrm{ACPW_1}}}}{480\pi K(k_s)K(k_3')}\,[\Phi(b-a,k_3)+\Phi(2a,k_3')] \tag{48}$$

where Φ is given by (30) and

$$k_3 = \sqrt{\frac{2a}{b+a}} \tag{49}$$

$$k_3' = \sqrt{1-k_3^2} = \sqrt{\frac{b-a}{b+a}} \tag{50}$$

where $2a$ is the strip width, $b-a$ the slot width.

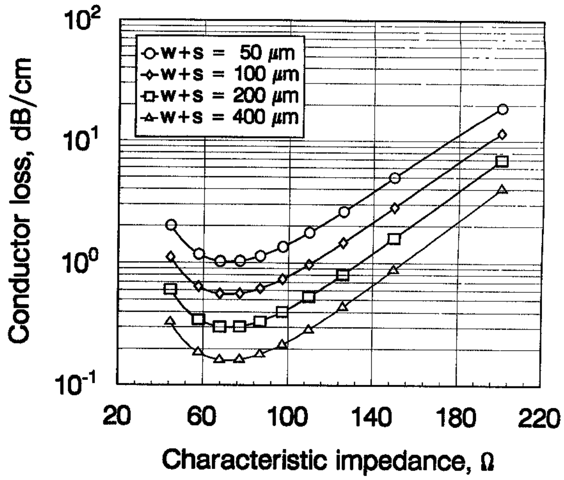

Fig. 11. Conductor loss of asymmetric coplanar waveguide with one lateral ground plane on semiinfinite GaAs substrate ($\epsilon_r = 12.8$) as a function of the characteristic impedance, for several values of the parameter $w+s$ (spacing from strip edge to ground plane). The frequency is 20 GHz, and a $5\mu m$ thick copper metallization was considered.

Although the approximate approach of (39) fails in this case, the partial capacitance technique can be now directly applied, owing to the absence of one of the lateral ground planes. The effective permittivity is expressed, according to the partial capacitance approximation, as

$$\epsilon_{\mathrm{eff}}^{\mathrm{ACPW_1}} = 1 + \frac{\epsilon_r - 1}{2}\frac{K(k_3)k(k_4')}{K(k_3')K(k_4)} \tag{51}$$

where

$$k_4 = \sqrt{\frac{\exp(2\pi a/h)-1}{\exp[\pi(b+a)/h]-1}} \tag{52}$$

$$k_4' = \sqrt{1-k_4^2} = \sqrt{\frac{\exp[\pi(b+a)/h]-\exp(2\pi a/h)}{\exp[\pi(b+a)/h]-1}}. \tag{53}$$

The above expression is equivalent to the one in [14, Section 13.4], but the parameters k_4 and k_4' are expressed in a somewhat simpler form.

The behavior of the conductor losses of the ACPW$_1$ as a function of the characteristic impedance is shown in Fig. 11 for several values of the strip-to-slot ratio. The removal of one of the lateral ground planes leads to an increase of the characteristics impedance, which in turn causes the impedance for minimum losses to be around 60–70Ω rather than around 50Ω, as for the CPW on a GaAs substrate.

Asymmetric coplanar lines with one ground plane are commonly used in electro-optic modulators. Recent quasi-static numerical analyses of the conductor losses of this structure on a LiNbO$_3$ substrate can be found in [3], [5], where results are presented on the skin-effect conductor attenuation as a function of the gap and strip widths. The analyses of [3], [5] accounts for the strip thickness, which plays a significant influence on both the characteristic impedance and the effective permittivity. In particular, this last parameter becomes geometry-dependent also for lines on semi-infinite substrates. In Fig. 12, results taken from [3, Fig. 4] are compared with the present analytical approximation of (48). While for the

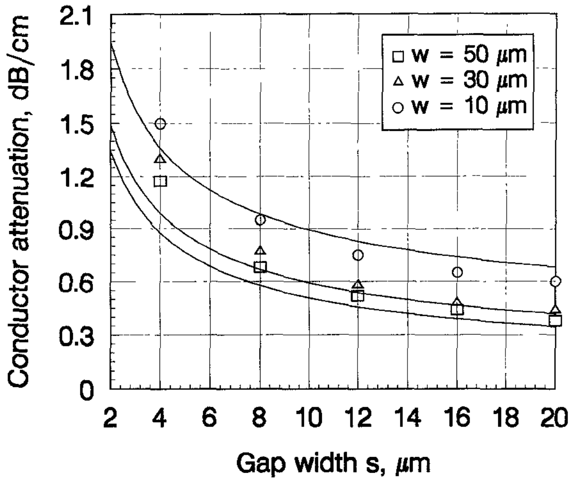

Fig. 12. Conductor loss of asymmetric coplanar waveguide with one lateral ground plane on semiinfinite $LiNbO_3$ substrate ($\epsilon_r = \sqrt{\epsilon_{11}\epsilon_{33}}, \epsilon_{11} = 28$, $\epsilon_{33} = 44$) as a function of the gap width s, for several values of the strip width w. The nominal frequency is 1 GHz, the strip thickness $t = 3\mu$m; an ideal gold metallization has been assumed ($R_s = 9.61 \times 10^{-3}\sqrt{f_{\text{GHz}}}\,\Omega$, [14]). The continuous line is from the present approach, symbols are from the thick-strip numerical approach of [3].

smallest gap widths the thin-strip approximation for the quasi-static parameters fails, and the discrepancy between the two approaches is significant, for larger gap and strip widths a satisfactory agreement results.[4] Similar remarks apply to a comparison with the results presented in [5, Fig. 5(b)], which is omitted for brevity.

D. General ACPS

For the ACPS, the partial capacitance technique is amenable to an exact implementation in terms of complete elliptic integrals. Therefore a unified set of expressions covers all cases: symmetric coplanar stripline (CPS) and the ACPS with finite-thickness or semi-infinite substrates. From the *in vacuo* attenuation expressed in (29) and (3), one has for the ACPS attenuation due to conductor losses, in natural units (Np/m):

$$\alpha_c^{\text{ACPS}} = \frac{R_s\sqrt{\epsilon_{\text{eff}}^{\text{ACPS}}}}{480\pi K(k)K(k')} \cdot \left[\Phi(b_1 - a, k) + \Phi(b_2 - a, k) + \Phi(2a, k') - \Phi(b_1 + b_2, k')\right] \tag{54}$$

[4] In [3] analytical approximations are provided for the parameters of the $ACPW_1$ with thick metallizations. These can be coupled to the present approach for the loss evaluation so as to yield a far better agreement with the results of [3, Fig. 4]. Since the thick case is of greater interest in the analysis of optical modulators, details are omitted for brevity and will be presented elsewhere.

where Φ is given by (30) and k is defined in (35).

The effective permittivity $\epsilon_{\text{eff}}^{\text{ACPS}}$ can be expressed, according to the partial capacitance approximation, as

$$\epsilon_{\text{eff}}^{\text{ACPS}} = 1 + \frac{\epsilon_r - 1}{2}\frac{K(k)K(k_5')}{K(k')K(k_5)} \tag{55}$$

where k_5 and k_5' are defined in (56), (57), respectively (see bottom of page).

The aforementioned expressions for the effective permittivity are equivalent to the expressions presented in [14, Section 13.5]. For symmetric lines $b_1 = b_2 = b$ and the above formulas can be reduced to equivalent forms [14], which are not reported here for the sake of brevity, through the use of (41)–(44).[5]

IV. Conclusions

CAD-oriented, analytical expressions have been proposed for the conductor losses of general asymmetric coplanar lines. The expressions hold for lines whose metallizations have thickness much smaller than the slot and strip widths, but suitably larger than the skin penetration depth at the operating frequency. Comparisons with numerical results concerning lines on low-ϵ_r and GaAs substrates suggest that, at least for thin lines and in the frequency range whereon the surface resistance model applies, the expressions for the conductor attenuation presented are accurate enough for design.

Appendix A
Evaluation of the Normalized Loss Factor

The normalized loss factor for the left-hand strip of an *in vacuo* ACPS, see (25), can be approximated by dividing the integration interval in three parts, corresponding to the strip (half) sides and to the strip top

$$\int_{z_1}^{z_2} P(z)|dz| = \int_{z_1}^{z_1'} P(z)|dz| + \int_{z_1'}^{z_2'} P(z)|dz| + \int_{z_2'}^{z_2} P(z)|dz|. \tag{58}$$

Concerning the *strip sides,* suitable approximations lead to the following results

$$\int_{z_1}^{z_1'} P(z)|dz| \approx \frac{1}{(z_2 - z_1)(z_3 - z_1)(z_4 - z_1)}$$

[5] The resulting expression for the effective permittivity of symmetric lines is different from the one adopted in [8], in which $\epsilon_{\text{eff}}^{\text{CPS}} \approx \epsilon_{\text{eff}}^{\text{CPW}}$ (see also [11, Ch. 7]). For the substrate thicknesses for which the partial capacitance approach holds, (55) in the symmetric case and the expression in [8] lead to similar results.

$$k_5 = \sqrt{\frac{\{\exp[2\pi(b_1 + a)/h] - \exp[2\pi(b_1 - a)/h]\}\{\exp[2\pi(b_1 + b_2)/h] - 1\}}{\{\exp[2\pi(b_1 + b_2)/h] - \exp[2\pi(b_1 - a)/h]\}\{\exp[2\pi(b_1 + a)/h] - 1\}}} \tag{56}$$

$$k_5' = \sqrt{1 - k_5^2}. \tag{57}$$

$$\cdot \int_{z_1}^{z_1'} \frac{1}{\sqrt{(z-z_1)\left(z_1^{\xi}-z\right)}}\, dz = \frac{\pi}{(z_2-z_1)(z_3-z_1)(z_4-z_1)}, \tag{59}$$

$$\int_{z_2'}^{z_2} P(z)|dz| \approx \frac{1}{(z_2-z_1)(z_3-z_2)(z_4-z_2)} \cdot \int_{z_2}^{z_2^{\xi}} \frac{1}{\sqrt{(z-z_2)(z_2'-z)}}\, dz = \frac{\pi}{(z_2-z_1)(z_3-z_2)(z_4-z_2)}. \tag{60}$$

The treatment of the *strip top*, corresponding to the interval $[z_1', z_2']$ is slightly more involved. Apart from two neighborhood of points $z_1'$and z_2' of dimension ε_1 and ε_2, $P(z)$ can be approximated by taking $z - z_i \approx z - z_i', i = 1, \cdots, 4$, as

$$P(z) \approx \frac{1}{(z-z_1)(z_2-z)(z_3-z)(z_4-z)} \equiv Q(z). \tag{61}$$

Since we have assumed that the strip thickness is small with respect to the strip width, we can take ε_1 to be much smaller than the strip width but suitably larger than $|z_1' - z_1|$ [21], and similarly for ε_2. In this way, as shown below, the integral is approximately independent of $\varepsilon_i, i = 1, 2$. Taking into account, (61), we can write

$$\begin{aligned}\int_{z_1'}^{z_2'} P(z)|dz| &= \int_{z_1'}^{z_2'} [P(z) + Q(z) - Q(z)]|dz| \\ &= \int_{z_1'}^{z_2'} Q(z)|dz| + \int_{z_1'}^{z_1'+\varepsilon_1} [P(z) - Q(z)]|dz| \\ &\quad + \int_{z_1'+\varepsilon_1}^{z_2'-\varepsilon_2} [P(z) - Q(z)]|dz| \\ &\quad + \int_{z_2'-\varepsilon_2}^{z_2'} [P(z) - Q(z)]|dz|. \end{aligned} \tag{62}$$

The first integral in the right-hand side can now be evaluated exactly as

$$\begin{aligned}\int_{z_1'}^{z_2'} Q(z)|dz| &= \frac{1}{(z_2-z_1)(z_3-z_1)(z_4-z_1)} \cdot \log\left(\frac{z_2'-z_1}{z_1'-z_1}\right) \\ &+ \frac{1}{(z_2-z_1)(z_3-z_2)(z_4-z_2)} \cdot \log\left(\frac{z_2-z_1'}{z_2-z_2'}\right) \\ &+ \frac{1}{(z_3-z_1)(z_2-z_3)(z_4-z_3)} \cdot \log\left(\frac{z_3-z_1'}{z_3-z_2'}\right) \\ &+ \frac{1}{(z_4-z_1)(z_4-z_2)(z_4-z_3)} \cdot \log\left(\frac{z_4-z_1'}{z_4-z_2'}\right), \end{aligned} \tag{63}$$

while for the other integrals we have

$$\int_{z_1'}^{z_1'+\varepsilon_1} [P(z) - Q(z)]|dz| \approx \frac{1}{(z_2-z_1)(z_3-z_1)(z_4-z_1)} \cdot \int_{z_1'}^{z_1'+\varepsilon_1} \left[\frac{1}{\sqrt{(z-z_1)(z-z_1')}} - \frac{1}{z-z_1}\right] dz \approx \frac{\log 4}{(z_2-z_1)(z_3-z_1)(z_4-z_1)} \tag{64}$$

since, for $\varepsilon_1 \gg |z_1' - z_1|$

$$\int_{z_1'}^{z_1'+\varepsilon_1} \left[\frac{1}{\sqrt{(z-z_1)(z-z_1')}} - \frac{1}{z-z_1}\right] dz = 2\log\left(\frac{\sqrt{\varepsilon_1 + z_1' - z_1} + \sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1 + z_1' - z_1}}\right) \approx \log 4. \tag{65}$$

Similarly, one has

$$\int_{z_2'-\varepsilon_2}^{z_2'} [P(z) - Q(z)]|dz| \approx \frac{\log 4}{(z_2-z_1)(z_3-z_2)(z_4-z_2)}. \tag{66}$$

Finally

$$\int_{z_1'+\varepsilon_1}^{z_2'-\varepsilon_2} [P(z) - Q(z)]|dz| \approx 0. \tag{67}$$

This completes the evaluation of the strip top contribution to the normalized loss factor, whose expression, omitted for brevity, can be obtained by substituting (64), (66), (67), and (63) into (62). By substituting the strip-top contribution and (59), (60), into (58), the normalized loss factor for the left-hand strip can be finally expressed as in (27), in which half thickness τ has been used instead of $z_1' - z_1$ and $z_2 - z_2'$ according to (18) and the approximations $z_i - z_j \approx \zeta_i - \zeta_j, i, j = 1, \cdots, 4$, have been made. The treatment of the right-hand strip is similar, and the final results is given in (28).

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